

# Single-photon Transistors Based on the Interaction of an Emitter and Surface Plasmons

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**Abstract** A symmetrical approach is suggested (Chang DE et al. Nat Phys 3:807, 2007) to realize a single-photon transistor, where the presence (or absence) of a single incident photon in a ‘gate’ field is sufficient to allow (prevent) the propagation of a subsequent ‘signal’ photon along the nanowire, on condition that the ‘gate’ field is symmetrically incident from both sides of an emitter simultaneously. We present a scheme for single-photon transistors based on the strong emitter-surface-plasmon interaction. In this scheme, coherent absorption of an incoming ‘gate’ photon incident along a nanotip by an emitter located near the tip of the nanotip results in a state flip in the emitter, which controls the subsequent propagation of a ‘signal’ photon in a nanowire perpendicular to the axis of the nanotip.

**Keywords** Single-photon transistor · Nanotip · Surface plasmon

## Introduction

The fundamental limit of a photonic transistor [1] is a single-photon transistor where the propagation of a single photon in the ‘signal’ field is controlled by the presence or absence of a single photon in the ‘gate’ field. Such a nonlinear device may find many interesting applications in fields such as optical communication [2], optical quantum computer [3], and quantum-information processing [4]. However, its physical realization is extremely demanding

because photons rarely interact. To achieve strong interaction between photons, several schemes based on either the resonantly enhanced nonlinearities of atomic ensembles [5–8] or individual atoms coupled to photons in cavity quantum electrodynamics (CQED) have been proposed [9–12]. Recently, a robust, practical approach based on the tight concentration of optical fields associated with guided surface plasmons (SP) on conducting nanowires has emerged [13]. However, this scheme works on condition that the optical ‘gate’ is split into two completely same parts and having them incident from both sides of the emitter simultaneously.

In this paper, we present a scheme for a single-photon transistor consisting of a nanotip, a nanowire, and an emitter. A single ‘gate’ photon propagating along a nanotip is coherently stored under the action of a classic control field, which results in an internal state flip in the emitter. This conditional state flip can change the propagation of a subsequent ‘signal’ photon traveling along the nanowire. In our scheme, the aforesaid condition can be released, the single ‘gate’ photon is incident from one side of the nanotip and travels toward the emitter which locates near the tip of the nanotip.

Recently, as a new scheme to achieve strong coupling between light and an emitter, surface plasmons which are propagating electromagnetic modes confined to the surface of a conductor-dielectric interface, have attracted intensive interests [13–21]. Surface plasmons can reduce the effective mode volume  $V_{eff}$  for the photons, thereby achieving a substantial increase in the coupling strength  $g \propto 1/\sqrt{V_{eff}}$ . An effective Purcell factor  $P \equiv \Gamma_{pl}/\Gamma' > 10^3$  in realistic systems may be achievable according to the theoretical results in [18, 22], where  $\Gamma_{pl}$  is the spontaneous emission rate into the surface plasmons (photons) and  $\Gamma'$  describes contributions from both emission into free space and

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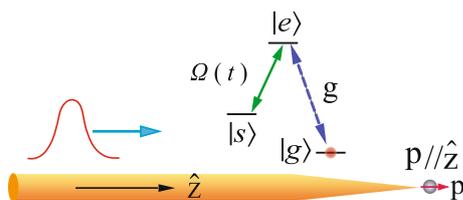
non-radiative emission via ohmic losses in the conductor. Furthermore, this strong coupling is broadband [13].

The propagation of surface plasmons can be significantly changed through interaction with a single emitter. For low incident powers, the reflection coefficient for an incoming photon of wavevector  $k$  is [13, 23]

$$r(\delta_k) = -\frac{1}{1 + \Gamma'/\Gamma_{pl} - 2i\delta_k/\Gamma_{pl}} \quad (1)$$

and the transmission coefficient  $t(\delta_k) = 1 + r(\delta_k)$ , where  $\delta_k \equiv c|k| - \omega_e$ . Here,  $c$  denotes the group velocity of the SPs and  $\omega_e$  is the energy difference between an excited state  $|e\rangle$  and a ground state  $|g\rangle$ . On resonance,  $r \approx -(1 - 1/P)$ , and thus the emitter in state  $|g\rangle$  works as a nearly perfect mirror for large  $P$ . The bandwidth  $\Delta\omega$  of the process determined by the total spontaneous emission rate  $\Gamma$  can be quite large. However, at high incident powers, the emitter rapidly saturates, as it cannot scatter more than one photon every time [13]. Two photons directly interact very weakly, but we can, first, let one photon change the state of an emitter, and then such change will significantly affect the propagation of another one. According to this principle a single-photon transistor may be realized physically [13].

First, we discuss the coherent storage of a single-photon in an emitter through a nanotip shown in Fig. 1. A three-level emitter is described by operator  $\sigma_{ij} = |i\rangle\langle j|$  ( $i, j = e, g, s$ ), with a ground state  $|g\rangle$ , a metastable state  $|s\rangle$ , and an excited state  $|e\rangle$ . The emitter is located along the  $z$ -axis of the nanotip and has a dipole moment  $\mathbf{p} = \langle e|\mathbf{er}|g\rangle$  parallel to the  $z$ -axis, which is a necessary condition for the strong interaction of an emitter and a nanotip [22]. State  $|s\rangle$  is decoupled from the surface plasmons owing to, for example, a different orientation of its associated dipole moment [13], but is resonantly coupled to the excited state  $|e\rangle$  via some classical, optical control field  $\Omega(t)$  with central frequency  $\omega_L$ . States  $|g\rangle$  and  $|e\rangle$  are coupled with strength  $g$  via the SP mode with wave vector  $k$  which is described by an annihilation operator  $a_k$ . States  $|g\rangle$ ,  $|s\rangle$ , and  $|e\rangle$  have the energy  $\omega_g = 0$ ,  $\omega_s$ , and  $\omega_e$ , respectively. The laser light satisfies the resonance condition:  $\omega_L + \omega_s = \omega_e$ . Since the



**Fig. 1** Schematic description of coherent storage of a single photon (SP) in the system consists of an emitter and a nanotip. The emitter is initially in the ground state  $|g\rangle$  and the dipole moment  $\mathbf{p}$  of the emitter is parallel to the axis of the nanotip. Under the action of the control field  $\Omega(t)$  dependent on the wave packet of the incoming photon, the capture of the incoming single photon may be realized while a state flip from  $|g\rangle$  to  $|s\rangle$  is induced

coupling  $g$  is broad-band, it can be assumed to be frequency independent [13, 22]. A linear dispersion relation  $\omega_k = c|k|$  is valid provided  $\hbar\omega_k < 2$  eV [21, 24]. Then, similar to the Hamiltonian in [13] describing the interaction of an emitter and a nanowire, the Hamiltonian for our model can be written in the form

$$H = \left( \omega_e - i\frac{\Gamma'}{2} \right) \sigma_{ee} + \omega_s \sigma_{ss} - \left( \Omega(t) e^{-i\omega_L t} \sigma_{es} + H.c. \right) + \int_{-\infty}^{\infty} dk c|k| a_k^\dagger a_k - \left( g \int_{-\infty}^{\infty} dk \sigma_{eg} a_k + H.c. \right), \quad (2)$$

where the emitter is assumed to be in the origin of the  $z$ -axis and the non-Hermitian term in  $H$  describes the decay of state  $|e\rangle$  at a rate  $\Gamma'$  into all other possible channels [18]. This effective hamiltonian holds under the condition that  $k_B T \ll \hbar\omega_e$ , e.g., if  $\hbar\omega_e = 1$  meV,  $T < 1$  K, where  $k_B$  is the Boltzmann constant [13].

The general time-dependent wave function for a system containing one excitation can be written in the form [13, 26]

$$|\psi(t)\rangle = \int_{-\infty}^{\infty} dk c_k(t) a_k^\dagger |g, vac\rangle + c_e(t) |e, vac\rangle + c_s(t) |s, vac\rangle, \quad (3)$$

where  $|vac\rangle$  denotes the vacuum state of the optical field. In the right-hand side of Eq. 3, the SP propagating toward (away from) the tip is described by that with  $k > 0$  ( $k < 0$ ). Under the Hamiltonian given in Eq. 2, the time evolution of coefficients  $c_k(t)$  and  $c_e(t)$  (in a rotating frame) is described by the following equations:

$$\dot{c}_k(t) = -i\delta_k c_k(t) + igc_e(t), \quad (4)$$

$$\dot{c}_e(t) = -\frac{\Gamma'}{2} c_e(t) + i\Omega(t)c_s(t) + ig \int_{-\infty}^{\infty} dk c_k(t). \quad (5)$$

Integrating Eq. 4 yields

$$c_k(t) = c_k(-\infty) e^{-i\delta_k t} + ig \int_{-\infty}^t dt' c_e(t') e^{-i\delta_k(t-t')}. \quad (6)$$

Substituting Eq. 6 into Eq. 5, in a way similar to the Wigner-Weisskopf theory of spontaneous emission [25, 13], we obtain the following equations for the atomic state amplitudes,

$$\dot{c}_e(t) = i\Omega(t)c_s(t) - \frac{\Gamma_{pl} + \Gamma'}{2} c_e(t) + i\sqrt{2\pi}gE_{in}(t), \quad (7a)$$

$$\dot{c}_s(t) = i\Omega^*(t)c_e(t), \quad (7b)$$

where  $\Gamma_{pl} = 2\pi g^2/c$  is the spontaneous emission rate into the SP modes and  $E_{in}(t) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} dk c_k(-\infty) e^{-i\delta_k t} =$

$1/\sqrt{2\pi} \int_0^\infty dk c_k(-\infty) e^{-i\delta_k t}$  is the incoming single-photon wave function (in a rotating frame), assuming that  $c_k(-\infty) = 0$  if  $k < 0$  for the incoming field.

Below we will show that, from Eq. 7, the amplitudes  $c_e(t)$  and  $c_s(t)$  including the control pulse  $\Omega(t)$  can be expressed in terms of  $E_{in}(t)$ . We assume that the photon storage process induces no outgoing field at the end, that is  $c_k(\infty) = 0$ , which combined with Eq. 6 yields

$$c_e(t) = \frac{icE_{in}(t)}{\sqrt{2\pi}g}. \tag{8}$$

From Eq. 7, we can solve for the amplitude of  $c_s(t)$ :

$$\frac{d}{dt} |c_s(t)|^2 = c |E_{in}(t)|^2 - \frac{c}{P} |E_{in}(t)|^2 - \frac{c}{\Gamma_{pl}} \frac{d}{dt} |E_{in}(t)|^2, \tag{9}$$

and the phase of  $c_s(t)$ :

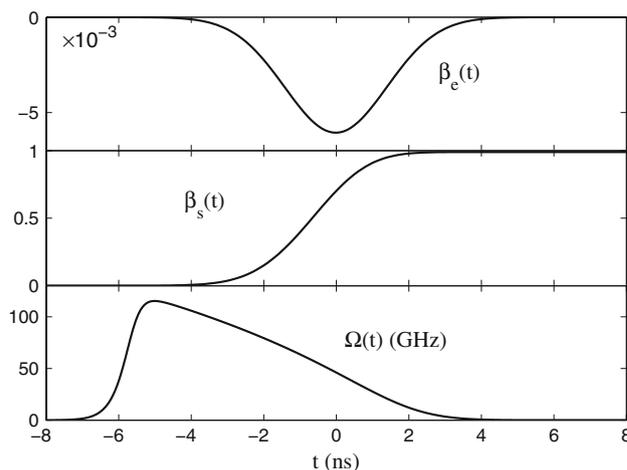
$$\frac{d\theta}{dt} = \frac{i}{|c_s(t)|^2} \left[ c_e(t) \left( \frac{d}{dt} c_e^*(t) + \frac{\Gamma_{pl} + \Gamma'}{2} c_e^*(t) + i\sqrt{2\pi}gE_{in}^*(t) \right) + \frac{1}{2} \frac{d}{dt} |c_s(t)|^2 \right]. \tag{10}$$

Then, from Eq. 7b, we can express  $\Omega(t)$  in terms of the amplitudes that have been solved above:

$$\Omega(t) = i \left( \frac{d}{dt} c_s^*(t) \right) / c_e^*(t). \tag{11}$$

Considering that the incoming field vanishes at  $t = \pm\infty$  and the normalization condition  $\int_{-\infty}^\infty dt |E_{in}(t)|^2 = 1/c$ , from Eq. 9, we have  $|c_s(\infty)|^2 = 1 - 1/P$ , which is physically equivalent to the probability for successful photon storage and spin flip from  $|g\rangle$  to  $|s\rangle$ . In the numerical simulation of a single-photon coherent storage (Fig. 2), we assume  $g = 1.6 \times 10^{10} \text{ m}^{1/2} \text{ s}^{-1}$ ,  $P = 100$ ,  $E_{in}(t) = i\sqrt{\frac{\sqrt{2}}{a\sqrt{\pi}}} e^{-(ct/a)^2} \text{ m}^{-1/2}$  with  $c = 1.5 \times 10^8 \text{ m/s}$  [19],  $a = 0.3 \text{ m}$ , and the emitter is initially in state  $|g\rangle$ . When this storage process finished,  $c_s(\infty) = 0.9950$ . If the incoming field contains no photon, the emitter is not affected by the control field  $\Omega(t)$  and remains in state  $|g\rangle$  for the whole process. Thus, when the control field  $\Omega(t)$  is turned off, the internal state of the emitter is  $|s\rangle(|g\rangle)$  provided the incoming field along the nanotip containing one (no) photon.

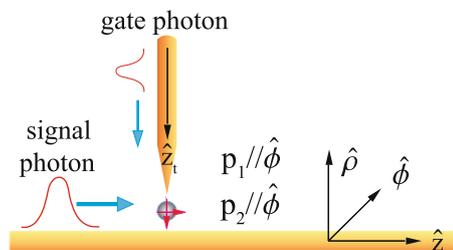
In our scheme for photon transistors, the emitter has such four energy levels, ground state  $|g\rangle$ , metastable state  $|s\rangle$ , and two excited states  $|e_i\rangle$  with energy  $\omega_i$  ( $i = 1, 2$ ) that the dipoles  $\mathbf{p}_1 = \langle e_1 | e\mathbf{r} | g \rangle \parallel \hat{\phi}$  and  $\mathbf{p}_2 = \langle e_2 | e\mathbf{r} | g \rangle \perp \hat{\phi}$  shown in Fig. 3, where  $\hat{\phi}$  is a unit vector oriented along the azimuthal axis (while  $\hat{z}$  is along the axis of the nanowire and  $\hat{\rho}$  is the unit vector oriented radially out). The nanotip is placed in such a way that the dipole moment  $\mathbf{p}_1$  located along the axis of the nanotip denoted by  $\hat{z}_t$  and oriented parallel to  $\hat{z}_t$ . We further assume that only the fundamental



**Fig. 2** Numerical simulation of an coherent storage of a single photon in the system of an emitter and a nanotip. (a) Amplitudes of the state  $c_{e1}$ . (b) Amplitude of the state  $\beta_{s1}$ . (c) The control field  $\Omega(t)$

surface plasmon mode of the nanotip and nanowire are excited surface plasmons [22].

In the stage of photon storage, the ‘gate’ photon propagating along the nanotip is on resonant with the transition  $|g\rangle \rightarrow |e_1\rangle$  and the frequency  $\omega_L$  of the control field  $\Omega(t)$  satisfies the resonance condition  $\omega_L + \omega_s = \omega_{e_1}$ . In this stage, the emitter does not excite the fundamental plasmon mode of the nanowire because  $\mathbf{p}_1 \parallel \hat{\phi}$  and  $\mathbf{p}_2$  is off resonant with the ‘gate’ field [22]. Thus, the aforesaid storage protocol can be applied to the system comprising the nanotip, the nanowire, and the emitter. In the second stage, the ‘signal’ field containing one photon resonant with the transition  $|g\rangle \rightarrow |e_2\rangle$  propagates along the nanowire. This field will not excite the fundamental plasmon mode in the nanotip since  $\mathbf{p}_2 \perp \hat{\phi}$  and  $\mathbf{p}_1$  is off resonant with the ‘signal’ field [22]. Thus, the propagating property of SPs can be used in this situation.



**Fig. 3** (Color online) Schematic picture of a single photon transistor. The nanotip is perpendicular to the nanowire. The emitter has four energy levels, with dipole moments  $\mathbf{p}_1 \parallel \hat{\phi}$  and  $\mathbf{p}_2 \perp \hat{\phi}$ . The single ‘gate’ photon propagating along the nanotip is coherently absorbed under the action of the control field  $\Omega(t)$ , which results a state flip from  $|g\rangle$  to  $|s\rangle$ . This conditional state flip can control the propagation of the ‘signal’ photon traveling along the nanowire

Combining the techniques of state-dependent conditional reflection and single-photon storage, a single-photon transistor can be realized [13]. First, the emitter is initialized in state  $|g\rangle$ . Under the action of the control field  $\Omega(t)$ , the presence or absence of a photon in a ‘gate’ pulse with frequency  $\omega_1$  traveling along the nanotip flips the internal state of the emitter to state  $|s\rangle$  or remains in state  $|g\rangle$  during the storage process. Then, this conditional flip can control the propagation of subsequent ‘signal’ photons with frequency  $\omega_2$  propagating along the nanowire. Thus, the interaction of subsequent signal pulse and the emitter depends on the internal state of emitter after the storage. If the emitter is in the state  $|g\rangle$ , the signal field is near, completely reflected by the emitter. Otherwise, the emitter is in the state  $|s\rangle$ , then the field is near-completely transmitted because  $|s\rangle$  does not interact with the surface plasmon. The storage and conditional spin flip makes the emitter either highly reflecting or completely transparent depending on the gate field containing none or one single-photon. Thus, the presence or absence of a single incident photon in a ‘gate’ field is sufficient to control the propagation of the subsequent ‘signal’ field, and the system therefore can serve as an efficient single-photon switcher or transistor.

As a summary, we have presented a scheme for a single-photon transistor, where the ‘gate’ field propagates along a nanotip and the ‘signal’ field travels along a nanowire perpendicular to the nanotip. A single ‘gate’ photon can control the propagation of a single ‘signal’ photon through changing the internal state of an emitter assisted by classic control field. This transistor may find many important applications in areas such as efficient single-photon detection [26] and quantum information science. Based on this scheme, the controlled-phase gate [9] for photons can be made; furthermore, a CNOT gate which is a key part of an optical quantum computer [3] is available. This system may also be a promising candidate for realizing electromagnetically induced transparency-based nonlinear schemes [5–8].

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